JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 40, No. 3, May–June 2003

Probabilistic Risk Assessment of Solid-Propellant Rocket Motors

Unyime O. Akpan* and Timothy E. Dunbar†
Martec Limited, Halifax, Nova Scotia B3J 3J8, Canada
and
Franklin C. Wong‡
Defence R&D Canada—Valcartier, Val-Belair, Quebec G3J 1X5, Canada

An approach is developed for probabilistic risk assessment of the propellant contained in solid rocket motors. Depending on the storage conditions, the propellant might be subjected to thermally induced loads that provoke mechanical damage. Time-dependent random functions for thermal loading are presented along with temperature-dependent and loading-rate-dependent structural capacity models. Limit state functions are formulated for two critical structural responses, namely, bore cracking and propellant/insulant debonding. Using first- and second-order reliability methods, instantaneous reliabilities are calculated and are used in progressive reliability computations. The sensitivities of the structural reliabilities to statistical and probabilistic descriptions of capacity and response input parameters are investigated. Example calculations for motor storage at various sites are used to demonstrate the methodology. Progressive reliability estimates are shown to be lower than instantaneous reliability predictions and a better indicator of motor service life. The major findings from sensitivity studies of the example problems here are that statistical values of propellant capacity (i.e., mean and standard deviation) have the greatest influence on predicted service life, whereas probabilistic distribution is least influential. Moreover, storage in an extremely cold environment has a much more significant effect on service life than does storage in moderately cold weather conditions. Thus, it can be concluded that reducing variability in propellant capacities through material processing will lead to significant improvements in the service life estimate of solid rocket motors.

Nomenclature

a	=	inner diameter of solid-rocket-motor (SRM) propellant
a_T	=	temperature-dependent shift factor
b	=	outer diameter of SRM propellant
C	=	time-dependent and load-rate-dependent propellant capacity
$E_p(t)$	=	propellant modulus
$F[x(t)]$	=	cumulative distribution function of the basic random variables at time t
$f[x(t)]$	=	joint probability density function of the basic random variables at time t
$G(t)$	=	time-dependent stress relaxation modulus
G_e	=	equilibrium modulus
$g(t)$	=	limit state function defining propellant failure
L	=	thermal storage induced propellant loading
$P_f(t)$	=	instantaneous failure probability at time t
T_{ref}	=	reference temperature
$T(t)$	=	environmentally induced temperature load
T_{sfree}	=	propellant's stress-free temperature
t	=	instantaneous time
α_c	=	thermal coefficient of expansion of the case
α_p	=	propellant's thermal coefficient of expansion
α^*	=	unit normal vector at the design point
β	=	reliability index
$\gamma_p(t)$	=	progressive or time-dependent reliability
$\Delta T(t)$	=	time-dependent change in temperature, °C
$\kappa_j(j = 1, 2, \ldots, n-1)$	=	ordered main curvatures of the original failure surface in the u space at the design point
λ	=	ratio of outer to inner case diameters
$\lambda(\xi)$	=	conditional probability function (hazard rate)
λ_i, G_i	=	Prony series coefficients
ν_c	=	Poisson ratio of the case
τ	=	reduced time
Φ	=	standard normal cumulative distribution function
Ω	=	failure domain defined by $\Omega = [g(t) < 0]$

Introduction

TACTICAL missiles using solid rocket motors (SRMs) can be stockpiled in a variety of geographical locations, ranging from hot and dry desertlike conditions to extreme cold conditions in the high arctic. Uncontrolled thermal environments induce random stresses and strains in the propellant of a case-bonded rocket motor. If the magnitude of the induced stress or strain exceeds the capacity of the propellant to support such a load, cracks or debonds can form and cause a catastrophic failure of the motor when it is fired.

The state of the propellant governs a solid rocket motor's service life. Propellant properties, however, are temperature and loading-rate dependent, thereby posing unique challenges to the assessment of the service life of SRMs. Research over the past several years[1–3] has shown that the initial values of mechanical and thermal properties of a propellant have high levels of variability and uncertainty. Furthermore, long-term storage of a motor under randomly varying thermal conditions leads to chemical aging, which, in turn, affects the values and variability of these mechanical properties. Therefore, any technique employed for estimating service life must account for these factors. Specifically, the technique must acknowledge the presence of uncertainties and variabilities and account for them in a rational and systematic manner. Probabilistic methods have been shown to be capable of meeting these challenges.

Several strategies for assessing the structural integrity of solid rocket motors have been developed over the years. They include deterministic approaches that are based on cumulative damage and estimates of margins of safety. Some of these approaches are summarized in Chapter 6 of Ref. 4. Simplified probabilistic techniques

Received 26 June 2002; revision received 2 December 2002; accepted for publication 11 January 2003.

*Senior Research Engineer, Suite 400, 1888 Brunswick Street. Member AIAA.
†Research Engineer, Suite 400, 1888 Brunswick Street.
‡Defence Scientist, Precision Weapons Section, 2459 Boulevard Pie XI North. Member AIAA.

have also been presented,[4,5] which define material capacities and stresses as normal random variables and employ either numerical integration or Monte Carlo simulation (MCS). However, these methods are either computationally expensive or do not fully account for uncertainties in the basic random variables such as propellant modulus, thermal expansion, etc.

The present study is focused on formulating and implementing a computationally efficient and robust probabilistic approach for quantifying the service life of SRMs under environmental loading conditions that fully account for uncertainties at the basic level. In the following sections limit state functions for assessing critical propellant responses are developed, and efficient instantaneous and progressive reliability solution strategies are formulated. Example computations are then presented to demonstrate the capabilities of the proposed methodology.

Limit State Functions for SRMs

Assessing the structural risk of a solid propellant requires the development of limit state functions with reference to critical structural responses, namely, propellant/insulant debonding and bore cracking.

The general form of the limit state model governing propellant/insulant debonding can be represented by

$$g_d(t) = C_d(t) - L_d(t) \tag{1}$$

where $C_d(t)$ is a function of the time and load-rate capacity of the bondline and $L_d(t)$ is a function of the effect of environmental storage load at the bondline. Equation (1) can be defined in terms of the radial strength capacity and the thermally induced radial stress at the bondline.

Similarly, the limit state function for bore cracking can be defined in a general manner by

$$g_c(t) = C_c(t) - L_c(t) \tag{2}$$

where $C_c(t)$ represents a model describing the time and load-rate capacity of the propellant and $L_c(t)$ denotes the effect of environmental storage load at the bore of the motor. Equation (2) can be defined either in terms of the propellant strength capacity and thermally induced circumferential stress at the bore or in terms of the circumferential bore strain and corresponding strain capacity.

A number of analytical, numerical, and probabilistic-based models exist with which to evaluate the structural response of SRMs. Four such models will be discussed in the following sections, including models for critical operating strains and stresses, thermal-loading models, and models used for assessing propellant strength and strain capacities.

Critical Operating Strain

Tactical rocket motors have bore configurations of varying complexity, ranging from simple cylindrical shapes to complex spoked bore geometries. Moreover, propellants of varying composition can be used along the longitudinal axis of a motor. Obviously, the selection of a structural model for studying the response of a motor depends largely on the complexity of the motor design. For the more complex designs use of finite element and/or analytical models might be required.[4]

In the current study a case-bonded motor with a cylindrical bore geometry is examined (Fig. 1). Plane strain conditions are assumed to exist at the midpoint of this motor, for which the critical strain occurs in the circumferential direction at the bore. This strain can be represented by Eq. (3)[2,6]:

$$L_c(t) = \varepsilon_\theta(t) = \log_e\{1 + \varepsilon'_\theta(t)\} \tag{3}$$

where

$$\varepsilon'_\theta(t) = \left(\tfrac{3}{2}\right)\alpha_R \lambda^2 \Delta T(t) \tag{4}$$

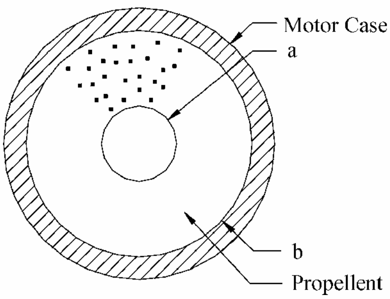

Fig. 1 Schematic diagram of case-bonded SRM.

$$\alpha_R = \alpha_p - \tfrac{2}{3}(1 + \nu_c)\alpha_c \tag{5}$$

$$\lambda = b/a \tag{6}$$

$$\Delta T(t) = T(t) - T_{\text{sfree}} \tag{7}$$

Critical Operating Stress

For a case-bonded SRM with a cylindrical bore, such as that considered herein (recall Fig. 1), two stresses are considered critical, namely, the radial stress at the case/grain interfacial bondline and the circumferential stress at the motor bore. Assuming once again that plane strain conditions exist at the midpoint along the motor, the bondline radial stress can be modeled analytically by[2]

$$L_b(t) = \sigma_r(t) = E_p(t)\alpha_R(\lambda^2 - 1)\Delta T(t) \tag{8}$$

and the circumferential bore stress can be represented by

$$L_c(t) = \sigma_{rc}(t) = 2E_p(t)\alpha_R\lambda^2\Delta T(t) \tag{9}$$

where

$$E_p(t) = 3G(t) \tag{10}$$

in which $G(t)$ represents the time-dependent stress-relaxation modulus

$$G(t) = G_e + \sum_{i=1}^{N} G_i e^{-\tau/\lambda_i} \tag{11}$$

$$\tau = \frac{t}{a_T} \tag{12}$$

$$\log a_T = \frac{-C_1(T - T_{\text{ref}})}{C_2 + (T - T_{\text{ref}})} \tag{13}$$

Environmental Thermal Loading Model

A probabilistic model has been successfully developed for environmental thermal loading SRMs,[7] wherein the maximum daily temperature is described by a seasonal sinusoidal variation about some mean level, with a superimposed daily or diurnal fluctuation. The following mathematical expression is used herein to describe the maximum daily temperature $T(t)$

$$T(t) = T_m + T_y \sin[(2\pi/365)(t - t_0)] + T_d \tag{14}$$

where T_m is the mean value of yearly temperature, T_y denotes the yearly amplitude of temperature, T_d is the peak value of daily temperature, and t_0 represents the start of the yearly temperature fluctuation.

Models for Structural Capacities

The strength and strain capacities of an SRM propellant depend on a number of factors, including temperature, loading rate, and age. These capacities are determined experimentally through tensile and bondline testing of representative propellant samples. Propellant data are typically measured under a wide range of temperature and crosshead displacement rates to reflect its thermorheological

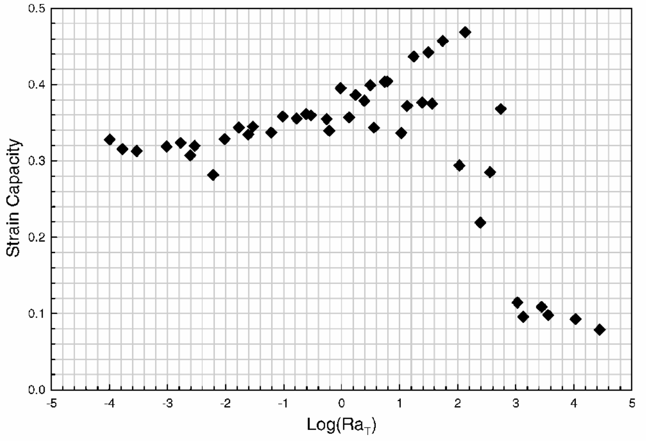

Fig. 2 Strain capacity as a function of log (Ra_T).

nature. Parameters of interest include the maximum stress, strain at maximum stress, and initial tensile modulus. A great deal of variability or scatter is typically observed in the strength and strain capacity data, thereby rendering probabilistic data characterization inevitable. Maximum stress and corresponding strain values are typically recorded as a function of reduced rate ($\log Ra_T$) and plotted in the log–log regime to produce failure envelopes for the propellant. Linear and piecewise nonlinear regression tools can then be used to calibrate the propellant capacities. An example of reduced rate strain data for a typical propellant is shown in Fig. 2.

Reliability Assessment Strategy

The reliability of the propellant in a rocket motor can be defined as the likelihood of the propellant to function according to its designed purpose for a particular time period. In this study, the goal is to calculate both instantaneous and progressive reliabilities, based on the critical performance functions defined in the preceding section.

Instantaneous Reliability

The instantaneous reliability of a rocket motor can be computed using the limit states defined in Eqs. (1) and/or (2), depending on whether the mode of failure is interfacial debonding or bore cracking. In either case the failure domain Ω is defined by a negative performance function {i.e., $\Omega = [g(t) < 0]$}, whereas its complement {$\Omega' = [g(t) > 0]$} defines the safe domain. The instantaneous failure probability at time t is defined by

$$P_f(t) = \int_\Omega f[x(t)]\,\mathrm{d}x \tag{15}$$

In general, the joint probability density function is unknown so evaluating the convolution integral is a formidable task. Several practical approaches have been developed, including first-order reliability methods (FORM) and second-order reliability methods (SORM).

First-order reliability methods, also known as fast probability integration schemes, are the most robust methodologies for computing instantaneous failure probability. The method uses the Hasofer–Lind formulation [or advanced first-order reliability method (AFORM) model], the basic concept of which involves the transformation of Gaussian (i.e., normal) random variables to the standard form (i.e., with zero mean and unit standard deviation). The Hasofer–Lind (or H-L) reliability index β_{HL} is then computed as the minimum distance from the origin to the limit state surface.

Although the H-L formulation is limited to cases involving Gaussian variables, the work represents an important milestone and has laid a solid foundation for the development of a class of procedures generically referred to as FORM. FORM procedures are optimization-based techniques that evaluate the reliability index β from which the failure probability P_f can then be computed using the relationship

$$\beta = \Phi^{-1}(P_f) \tag{16}$$

FORM procedures utilize the full distribution information of the random variables involved in the definition of the limit state function. Correlation between the variables is permitted.

Several techniques are available with which to complete FORM calculations. It is sufficient, however, to illustrate the basic features of the entire class via a description of a particular scheme called the HL-RF algorithm. This algorithm is named after Hasofer and Lind,[8] based on the work just described, and Rackwitz and Fiessler,[9] who first proposed the generalization of the H-L scheme to non-Gaussian random variables. The Hasofer–Lind and Rackwiz–Fisseler (HL–RF) algorithm is one of the most popular FORM procedures. The essential steps involved in FORM algorithms include 1) transformation of the vector of basic random variables from the original X space to the standard normal u space; 2) a search (usually in u space) for the point u^*, which is popularly referred to as the design point, failure point, or the most probable point (MPP), on the limit state surface [i.e., $g(u) = 0$] that has the highest joint probability density; 3) an approximation at the MPP of the failure surface in u space; and 4) a computation of the distance from the origin to the MPP called β, from which the failure probability P_f can be computed.

The transformation from the original (X space) to the standard normal (u space) is usually denoted by the transformation operator T such that

$$U = T(X) \tag{17}$$

This probability transformation scheme has been verified to yield extremely accurate results in reliability analysis.

The search for the MPP is conducted by means of the solution of an optimization problem. The optimization problem pertaining to the calculation of the Hasofer–Lind reliability index in u space can be summarized as follows.

Minimize:

$$D = \sqrt{u_i^T u_i} = \beta$$

Subject to:

$$g(u_i) = 0 \tag{18}$$

The solution of this problem locates the MPP, and the n-dimensional position vector of this point U^* is given by

$$U^* = \alpha^* \beta \tag{19}$$

where α^* denotes the unit normal vector at the MPP, that is,

$$\alpha^* = \frac{\nabla g(U^*)}{|\nabla g(U^*)|} \tag{20}$$

in which ∇ represents the gradient operator. The computed reliability index β has a one-to-one nonlinear relationship with the failure probability.

The HL-RF algorithm is currently the most widely used method for solving the constrained optimization problem in structural reliability.[10] The method is based on the following recursive formula:

$$U_{k+1} = \left[1\big/\nabla g^T(U_k)\nabla g(U_k)\right]\left[\nabla g^T(U_k)U_k - g(U_k)\right]\nabla g(U_k) \tag{21}$$

Experience shows that, for most situations, the HL-RF algorithm converges rapidly. It is the primary reliability methodology employed in the present study.

SORMs involve the use of a quadratic approximation of the performance function, which, in the event of a truly nonlinear limit state surface, generally provides slightly improved results over those predicted using FORM. Several algorithms have been developed for SORM, including the following:

1) Breitung (1984)[11]:

$$P_f = \Phi(-\beta)\prod_{j=1}^{n-1}(1+\beta\kappa_j)^{-\frac{1}{2}} \tag{22}$$

2) Hohenbichler (1988) (see Ref. 12):

$$P_f = \Phi(-\beta)\prod_{j=1}^{n-1}\left[1+\frac{\phi(\beta)}{\Phi(\beta)}\kappa_j\right]^{\frac{1}{2}} \tag{23}$$

3) Tvedt (1990)[13]:

$$P_f = \Phi(-\beta)R_e\left[i\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\int_{t=0}^{\infty}\frac{1}{t}e^{(t+\beta)^2/2}\prod_{j=1}^{n-1}(1-t\kappa_j)^{-\frac{1}{2}}\,\mathrm{d}t\right] \tag{24}$$

In Eqs. (22–24), β represents the FORM-based reliability index, and n denotes the number of random variables (X). Alternatively, the MCS technique, in which the failure set $g(X)$ is populated through generation of random samples, has proven to be a valuable instrument in reliability analysis. All of these techniques have been implemented into the COMPASS probabilistic analysis software,[14] which was employed throughout the course of this investigation.

Progressive Reliability

In the presence of a continuously varying environmental load, the instantaneous probability of failure is a function of time. The instantaneous failure probability at any time t is defined by $P[g(t) < 0]$, without regard to survival of a motor during previous years, and can be obtained by using Eq. (15).

Successive daily loadings are interdependent events and must therefore be appropriately accounted for in reliability estimation. Their interdependence is best accounted for through the use of time-dependent or progressive reliability estimates, which are based on conditional probability theory. In the present study the hazard rate or failure function strategy is employed. The progressive or time-dependent reliability $\gamma_p(t)$ of a stored missile structure is given by

$$\gamma_p(t) = \exp\left[-\int_0^t \lambda(\xi)\,\mathrm{d}\xi\right] \tag{25}$$

where ξ is simply the variable of integration and $\lambda(\xi)$ represents the hazard rate.[15,16] The hazard rate $[\lambda(t)]$ is defined as the probability of failure between time t and time $t + \mathrm{d}t$, given that no failure has occurred up to time t. For continuous systems, Ang and Tang[17] define the hazard rate as

$$\lambda(t) = \frac{f(t)}{1 - F(t)} \tag{26}$$

where $f(t)$ denotes the joint probability density function and $F(t)$ represents the joint cumulative distribution function. For discrete space with a daily increment, the hazard function becomes

$$\lambda(t_i) = \frac{P_f(t_i)}{1 - \sum_{j=1}^{i-1} P_f(t_i - 1)} \tag{27}$$

Substituting Eq. (27) into Eq. (25) gives the time-dependent reliability. The time-dependent failure probability is given by

$$P_{\mathrm{ft}}(t) = 1 - \gamma_p(t) \tag{28}$$

where the subscript ft denotes time-dependent failure probability. Equation (28) is used to estimate the progressive or time-dependent reliability. It is emphasized that $P_{\mathrm{ft}}(t) = 1 - \gamma_p(t)$ is not equivalent to $P[C(t) < L(t)]$, the latter being only an instantaneous failure at time t without regard to previous or future performance.

Demonstration of Method

To illustrate the proposed method, the service life of a case-bonded rocket motor with a cylindrical bore is predicted here. In this example reliability predictions will be based on the potential for bore cracking caused by excessive strain. The values of the Prony-series coefficients and the shift-factor parameters for the propellant are given in Table 1 and as follows: C_1, 5.22; C_2, 164.9; and T_{ref}, 297.0. Table 2 presents the probabilistic and statistical values of the random variables for this problem. Figure 2 presents the propellant's strain capacity as a function of reduced rate $\log(Ra_T)$. Figure 3 outlines the strategy for assessing the risk-based service life of the motor. Two storage sites, representing moderately cold and extremely cold environments, are investigated here. Plots of the average daily temperatures for the two sites are presented in Fig. 4.

Table 1 Prony-series coefficients for SRM propellant

i	G_i, Pa	λ_i, s
1	77.4520×10^6	3.2967×10^{-9}
2	40.0178×10^6	2.7109×10^{-8}
3	21.2522×10^6	2.2291×10^{-7}
4	14.8369×10^6	1.8330×10^{-6}
5	10.9891×10^6	1.5073×10^{-5}
6	7.9458×10^6	1.2394×10^{-4}
7	4.8011×10^6	1.0192×10^{-3}
8	2.5918×10^6	8.3804×10^{-3}
9	1.3000×10^6	6.8912×10^{-2}
10	6.6377×10^5	5.6667×10^{-2}
11	3.2938×10^5	4.6596×10^0
12	1.9124×10^5	3.8315×10^1
13	6.4188×10^4	3.1506×10^2

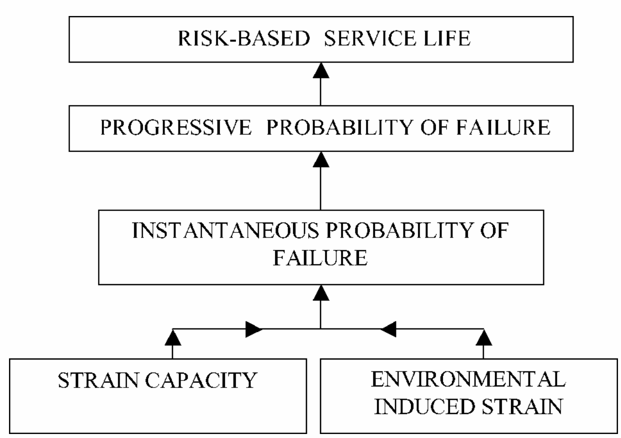

Fig. 3 Strain-based progressive reliability of solid rocket motors.

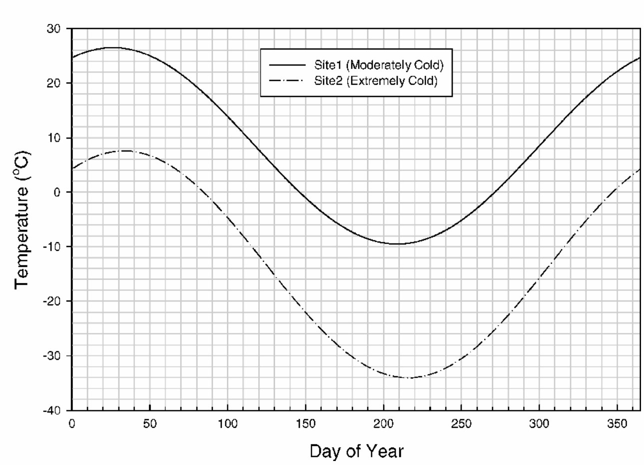

Fig. 4 Average value of daily temperature at sites 1 and 2.

Table 2 Mean value and COV of random variables used in analysis

Random variable	Distribution type	Mean value	COV
Strain capacity	Normal	Obtained from Fig. 2	0.1
Outside diameter b, in.	Fixed	3.21	——
Inside diameter a, in.	Fixed	1	——
Coefficient of thermal expansion of missile case α_c, °C	Lognormal	2.36×10^{-5}	0.1
Coefficient of thermal expansion of propellant α_p, °C	Lognormal	8.50×10^{-5}	0.1
Poisson ratio of case ν_c	Lognormal	0.33	0.1
Stress-free temperature T_{sfree}, °C	Fixed	60	——
Daily mean temperature T_m, °C	Fixed	6.00 (site 1)	——
		−12.88 (site 2)	——
Yearly amplitude of temperature T_y, °C	Fixed	18.0 (site 1)	——
		20.8 (site 2)	——
Peak value of daily temperature T_d, °C	Normal	2.50 (site 1)	2.0
		−0.35 (site 2)	1.0
Equilibrium modulus G_e, MPa	Lognormal	18	0.1

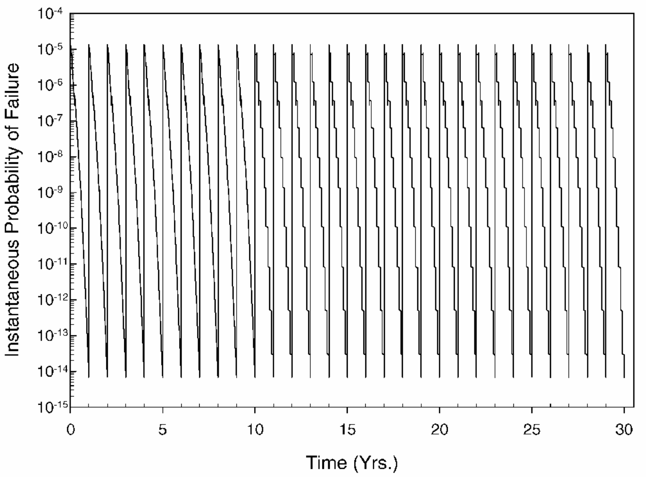

Fig. 5 Instantaneous probability of failure at site 2 (extremely cold).

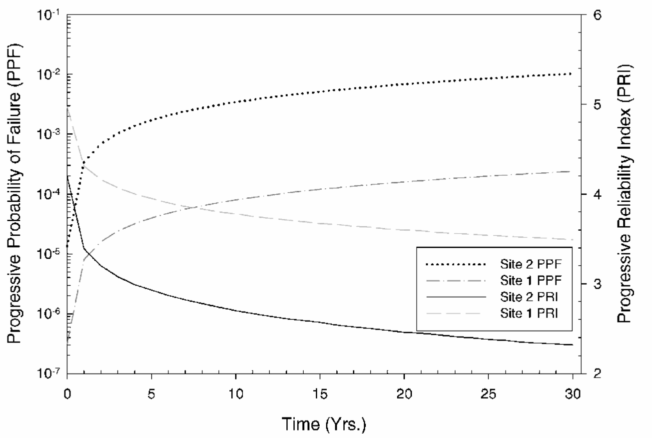

Fig. 6 Comparison of progressive probability of failure (PPF) and progressive reliability index (PRI) at sites 1 and 2.

Preliminary deterministic analyses were executed to compute the daily margin of safety at each site. The range of deterministic-based daily margins of safety for the two sites is presented here: site 1, 7.77–3.09; site 2, 4.52–1.96. It is shown that the extremely cold site (site 2) generally yields smaller safety margins than does the moderately cold site (site 1).

Instantaneous values of margin of safety for the two sites were computed using both SORM and MCS. Results show that although both methods produce comparable results (SORM, site 1, 1.250×10^{-6}; SORM, site 2, 4.270×10^{-4}; MCS, site 1, 1.249×10^{-6}; MCS, site 2, 4.268×10^{-4}) the SORM approach is at least 1000 times more computationally efficient than MCS (SORM, site 1, 4.52 s; SORM, site 2, 4.67 s; MCS, site 1, 6723 s; MCS, site 2, 5124 s). The instantaneous failure probability for the extremely cold site is presented in Fig. 5. For the moderately cold site (not shown) the instantaneous failure probability was an order of magnitude smaller.

Progressive failure probabilities and reliability indices for the two sites are presented in Fig. 6. The results indicate that the progressive failure probabilities are always higher than instantaneous failure probabilities. Moreover, progressive failure probabilities at the extremely cold site are always higher than those at the moderately cold site. Similarly, the corresponding progressive reliability indices at site 1 (moderately cold) are significantly higher than those at site 2 (extremely cold). Thus, for the motors considered in this study, it can be concluded that those stored in environments similar to the moderately cold site will have longer service lives. A nonconservative estimation of service life would result if instantaneous failure probabilities were used to assess risk.

Importance factors reflect the effect of uncertainty in input random variables on estimates of failure probability. The importance factor breakdowns for the random variables employed in the present study are shown in Figs. 7 and 8 for the moderately and extremely cold environments, respectively. It can be seen that uncertainty in the propellant-related properties, namely, strain capacity and thermal expansion coefficient, has the greatest impact on the predicted service life. Therefore, adequate resources should be devoted to accurate probabilistic and statistical calibration of these parameters. In contrast, uncertainty in random variables such as Poisson's ratio and the coefficient of thermal expansion of the motor case has only minimal effect on service life. As such, these parameters can be adequately modeled as fixed-value variables represented by their mean value.

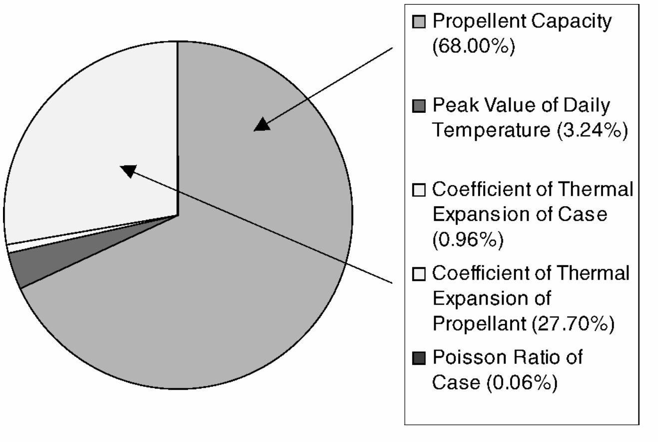

Fig. 7 Importance factors of random variables used in analysis for site 1 (moderately cold).

Sensitivities to probabilistic descriptions of the propellant strain capacity at both site 1 (moderately cold) and site 2 (extremely cold) were obtained in the present study. Because the results of

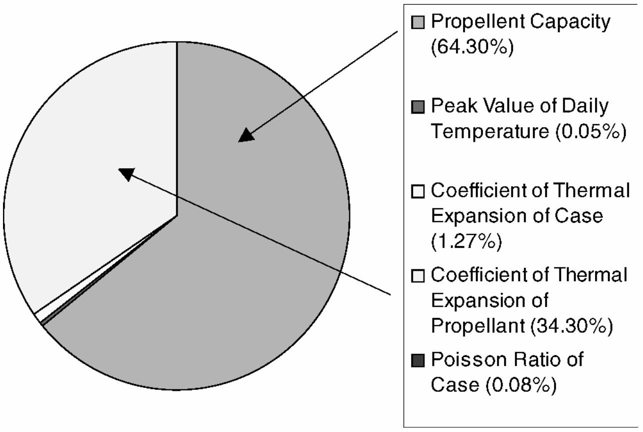

Fig. 8 Summary of importance factors of random variables used in analysis for site 2 (extremely cold).

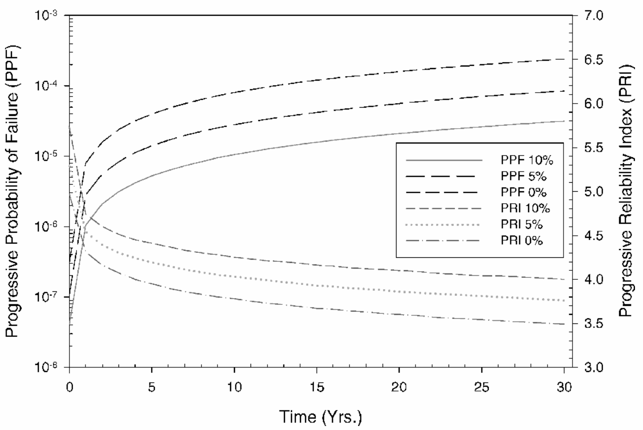

Fig. 9 Sensitivity of PPF and PRI to an increase in the mean value of propellant strain capacity at site 1.

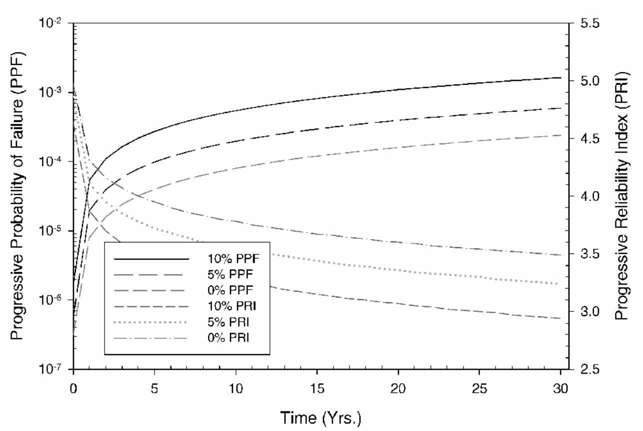

Fig. 10 Sensitivity of PPF and PRI to an increase in the coefficient of variation of propellant strain capacity at site 1.

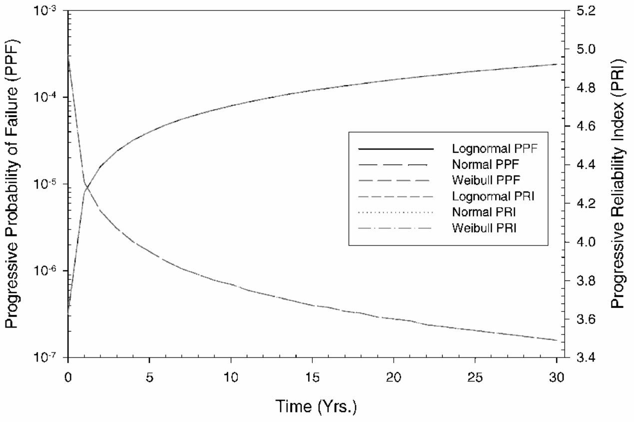

Fig. 11 Sensitivity of PPF and PRI to the probability distribution for propellant strain capacity at site 1.

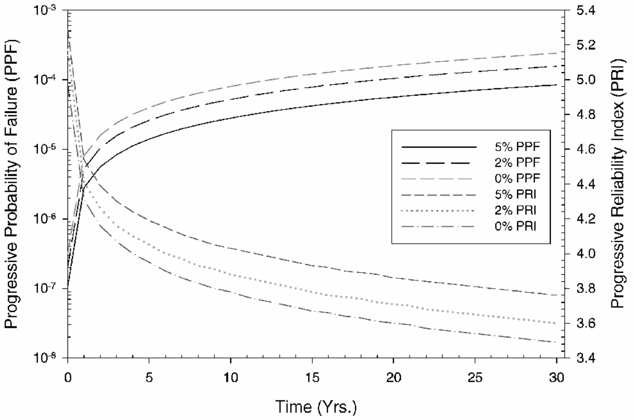

Fig. 12 Sensitivity of PPF and PRI to the estimation error in propellant strain capacity at site 1 (+ percentage signifies increasing strain capacity).

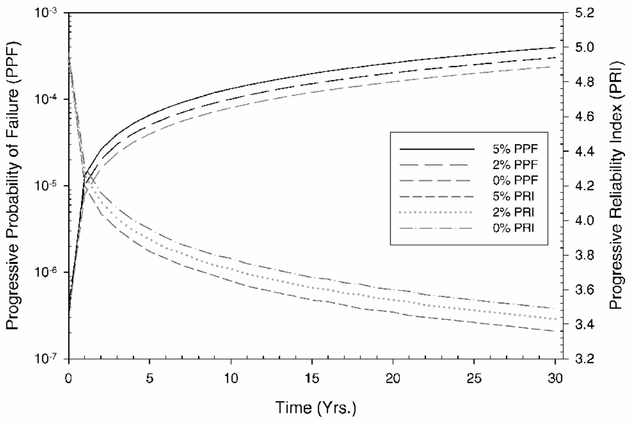

Fig. 13 Sensitivity of PPF and PRI to the estimation error in daily temperature at site 1 (+ percentage denotes underestimation of actual temperature).

the investigations at both sites are similar, only those for the moderately cold site (site 1) are illustrated in Figs. 9–11. The results indicate that the progressive probabilities of failure (PPF) and reliability indices (PRI) are very sensitive to the mean value and standard deviation [and thus, coefficient of variation (COV)] of the propellant strain capacity (Figs. 9 and 10). In contrast, Fig. 11 indicates that PPF and PRI levels are relatively insensitive to its probabilistic distribution type. These results suggest that efforts should be directed toward accurate statistical description of these parameters (i.e., mean value and standard deviation) rather than the probabilistic distribution (normal, lognormal, or Weibull, for example).

The impact of estimation error in key input parameters for probabilistic service life estimation, namely, propellant capacity and thermal load (temperature), was studied by using Monte Carlo simulation. Results for various levels of estimation error are presented in Figs. 12 and 13. They show that progressive probability-of-failure levels decrease with overestimation of the measured strain capacity, whereas a similar overestimation yields an increase in the corresponding progressive reliability index. An underestimation of the daily temperature acting on the motor causes an increase in progressive failure probability and a corresponding reduction in progressive reliability index. Comparing Figs. 12 and 13, it is seen that the error in measured capacity has more of an impact on the predicted service life than does an equal error in the measured thermal load. Therefore, as already shown by the importance factor analysis, more resources should be directed towards improving the procedures by which propellant capacities are characterized because the data will greatly influence the predicted service life of the rocket motor.

Limitations of the Method

A number of limitations have been identified in the current investigation. They can be summarized as follows: First, the structural analysis model used for the current study is analytical in nature and is restricted to case-bonded motors possessing a cylindrical bore. Also, the propellant is assumed to behave in a linear viscoelastic manner. As such, the model is not applicable to motors having complex bore geometries or propellants that exhibit nonlinear material behavior. Such complexities can be better suited to an analysis that employs stochastic finite element methods. Second, the probabilistic distribution and standard deviation of the propellant capacity have been assumed constant and independent of loading rate and temperature. This assumption might not be applicable to some propellants. Furthermore, the mean value of the propellant capacity is taken as that for a virgin material. In reality, however, this value is a function of cumulative damage. The impact of this kind of mechanical aging is not fully understood and has not been incorporated into the models. Finally, the effects of chemical aging on the propellant have not been accounted for in the current analysis. This fact is important because it is well accepted that the propellant response and failure properties are significantly affected by this phenomenon.

Efforts are currently underway to devise both experimental and numerical strategies that can overcome some of these limitations.

Conclusions

An approach for probabilistic risk assessment of SRMs subjected to environmental storage loads has been advanced. Time-dependent random functions for thermal loading have been presented along with temperature-dependent and loading-rate-dependent models for propellant and bondline capacities. Limit state functions were formulated for critical responses, namely, bore cracking and propellant/insulant debonding. First- and second-order reliability methods, as well as Monte Carlo simulation, were used to calculate instantaneous reliabilities of the rocket motor. Methodology for computing the progressive reliability of a rocket motor was also developed, and the service life of a motor stored at a moderately cold and an extremely cold site was investigated to demonstrate the proposed method. In addition, limitations of the proposed analysis method were summarized.

The results of the present study show that progressive reliability estimates are, in general, always higher than instantaneous reliability predictions and are a better indicator of motor service life. The importance factor and sensitivity studies indicated that statistical values of propellant capacity (i.e., mean and standard deviation) have the greatest influence on the predicted of service life. The service life estimate is, however, insensitive to the type of probabilistic distribution that describes the capacity data. Furthermore, the results predict that motor storage in an extremely cold environment would result in a shorter service life.

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M. P. Nemeth
Associate Editor